

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad \begin{cases} I_1 = g_{11}V_1 + g_{12}I_2 \\ V_2 = g_{21}V_1 + g_{22}I_2 \end{cases}$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad \begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases}$$

$$ABCD = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$

(1) $ABCD \rightarrow H$.

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0}$$

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$

$$\Rightarrow \text{for } V_2=0: \begin{cases} V_1 = -BI_2 \\ I_1 = -DI_2 \end{cases} \Rightarrow h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = \frac{-BI_2}{-DI_2} = \frac{B}{D}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} = \frac{I_2}{-DI_2} = -\frac{1}{D}$$

\Rightarrow for $I_1=0$: $CV_2 - DI_2 = 0$

$$\Rightarrow h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{C}{D}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} = \frac{AV_2 - BI_2}{V_2} = \frac{AV_2 - B(\frac{C}{D})V_2}{V_2} = \frac{AD - BC}{D}$$

$$\Rightarrow H = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

(2) $ABCD \rightarrow G$

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0}$$

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0}$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0}$$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0}$$

$$\Rightarrow \text{for } I_2=0: \begin{cases} V_1 = AV_2 \\ I_1 = CV_2 \end{cases} \Rightarrow g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0} = \frac{CV_2}{AV_2} = \frac{C}{A}$$

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0} = \frac{V_2}{AV_2} = \frac{1}{A}$$

\Rightarrow for $V_1=0$: $AV_2 = BI_2$

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0} = \frac{B}{A}$$

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0} = \frac{CV_2 - DI_2}{I_2} = \frac{C(\frac{B}{A})I_2 - DI_2}{I_2} = \frac{BC - AD}{A}$$

$$\Rightarrow G = \begin{bmatrix} \frac{C}{A} & \frac{BC-AD}{A} \\ \frac{1}{A} & \frac{B}{A} \end{bmatrix}$$